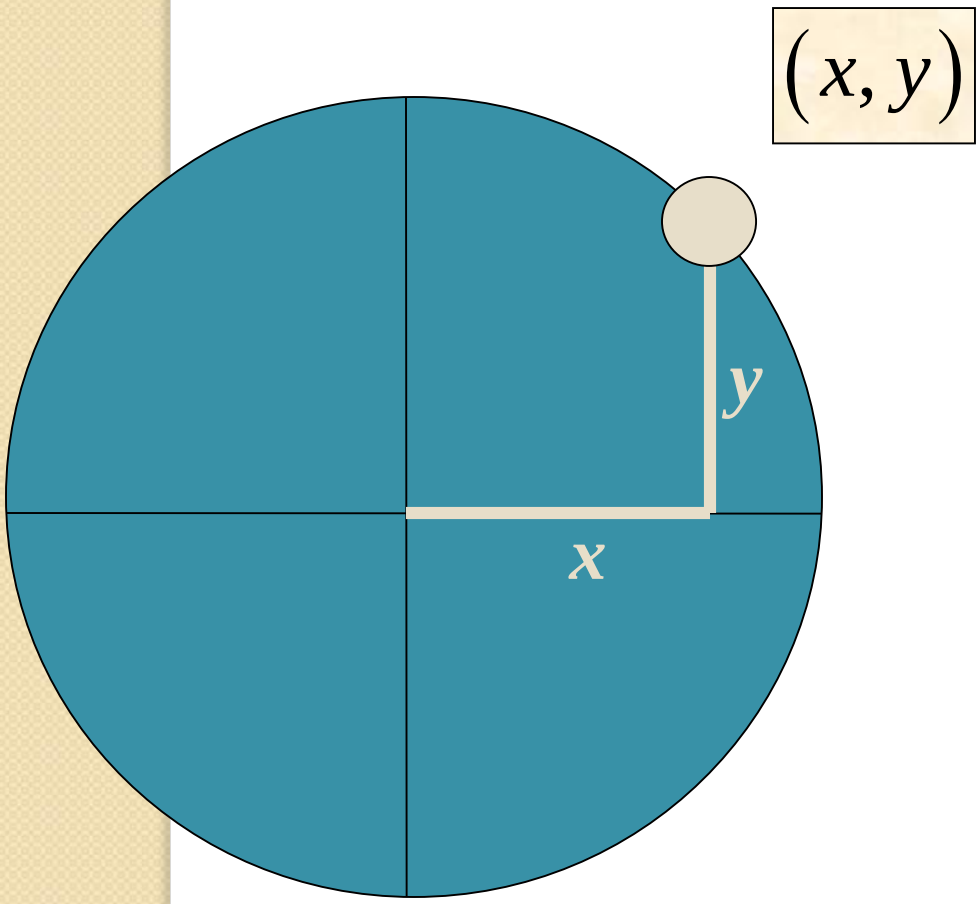
Polar Functions

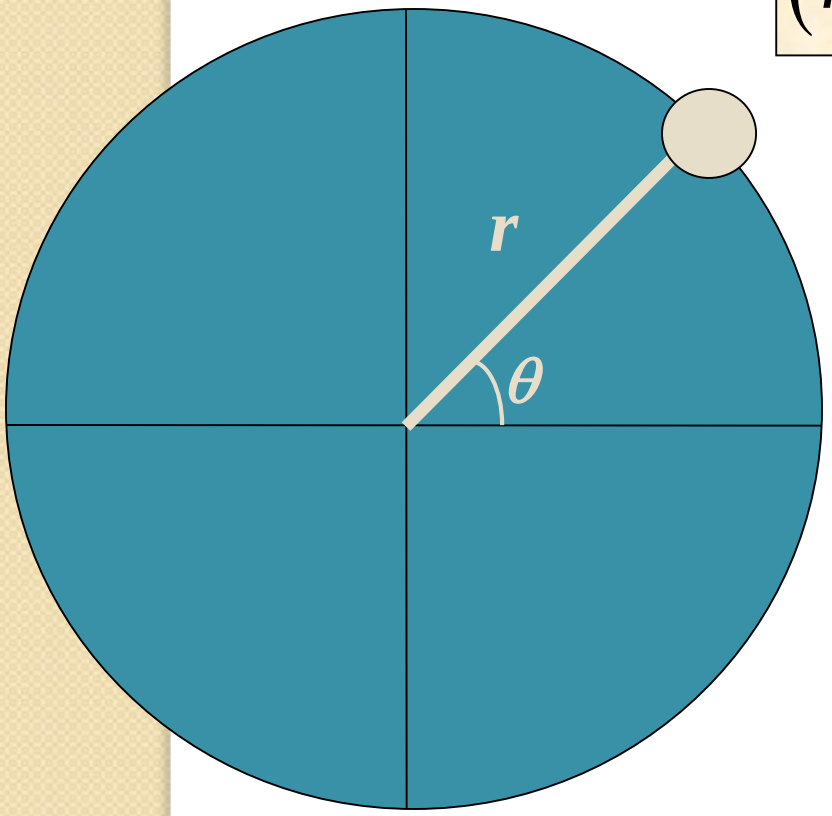
Section 9.4

Most of the time we graph on the Cartesian Plane.



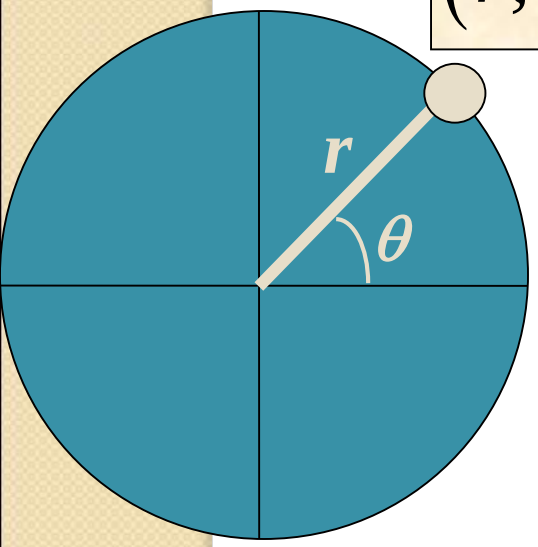
You can name the same point using Polar Coordinates

$$(r, \theta)$$

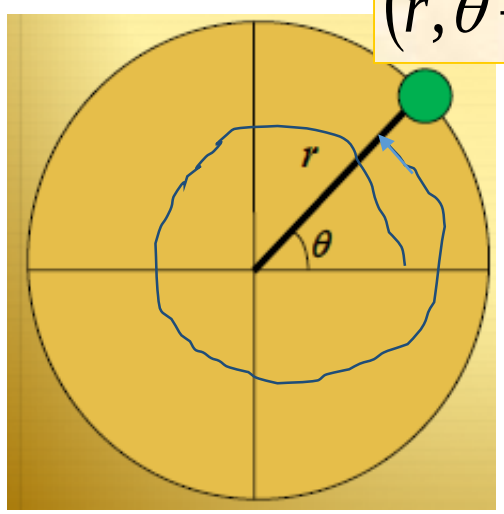


Each point can have many sets of polar coordinates.

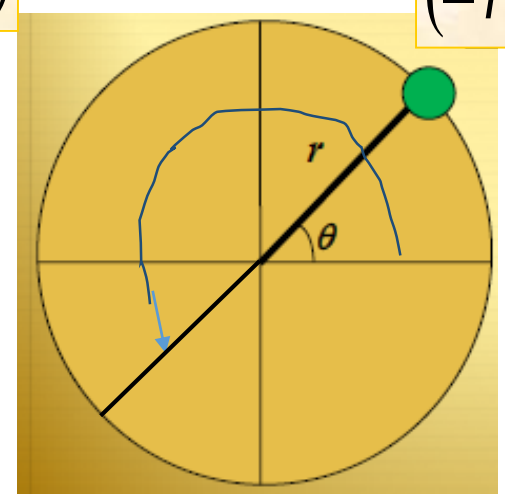
(r,θ)



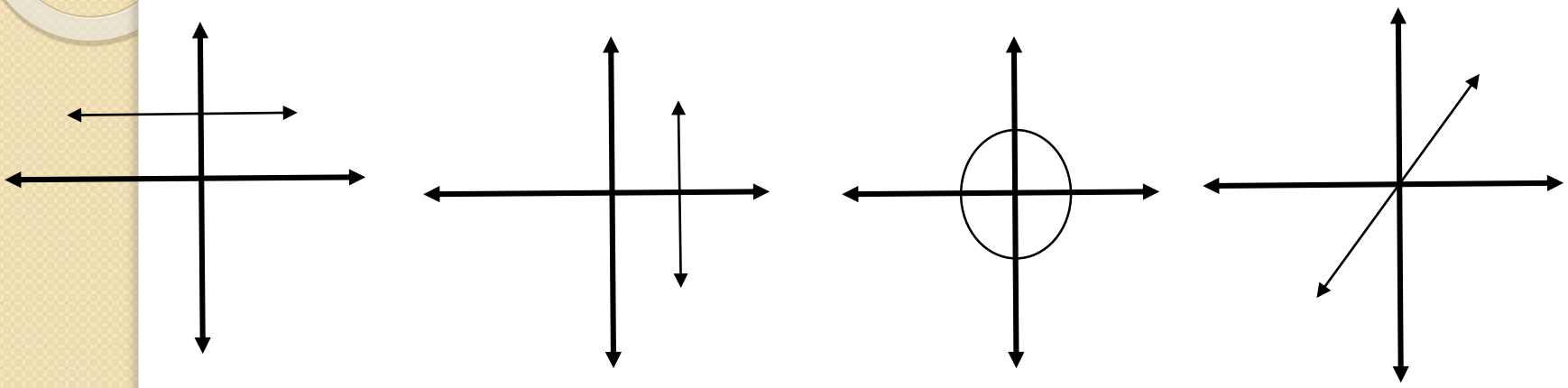
$(r,\theta+2\pi)$



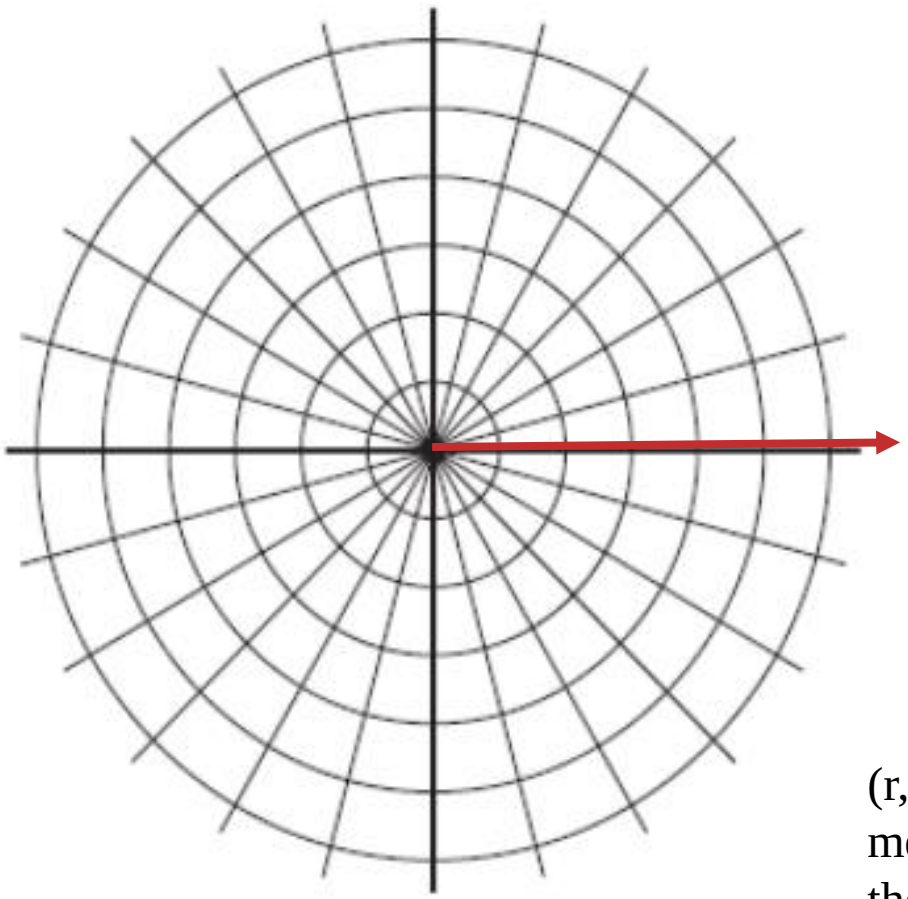
$(-r,\theta\pm\pi)$



Some functions are easier in Cartesian, some are easier in Polar



$y = 2$ $\quad$ $x = 2$ $\quad$ $r = 2$ $\quad$ $\theta = \frac{\pi}{4}$

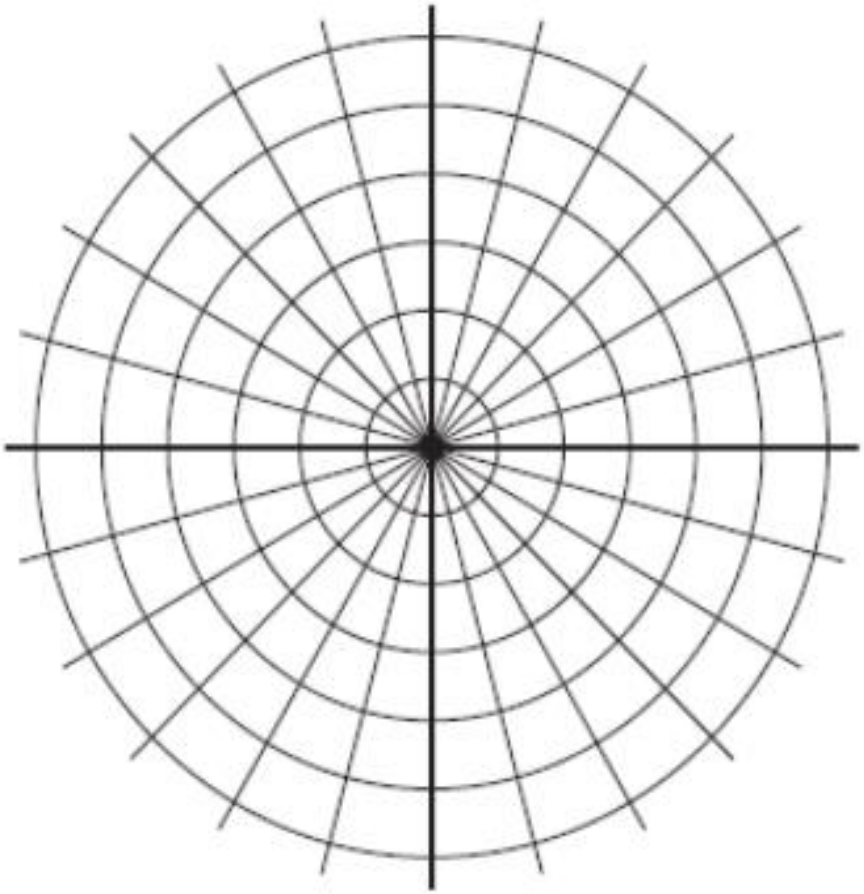


Graph the following points

Grid #1	Grid #2
A(2, π)	C(1, - π/6)
B(-3, π/2)	D(-3, 5π/6)

Grid #3	Grid#4
E(2, 2π/3)	G(-2, 5π/4)
F(-1, π/4)	H(-3, 0)

(r, θ) find angle direction first measured from the **polar axis**, then count radius forward(+) or backward (-) from the **pole**



Graph the following

<u>Grid #5</u>

$r = -1$

<u>Grid #6</u>

$\theta = \pi/3$

<u>Grid #7</u>

$$\begin{cases} 1 \le r \le 4 \\ 0 \le \theta \le \pi/2 \end{cases}$$

<u>Grid #8</u>

$$\begin{cases} r \le 0 \\ \theta = \pi/4 \end{cases}$$

<u>Grid #9</u>

$$\begin{cases} -3 \le r \le 2 \\ \theta = \pi/4 \end{cases}$$

<u>Grid #10</u>

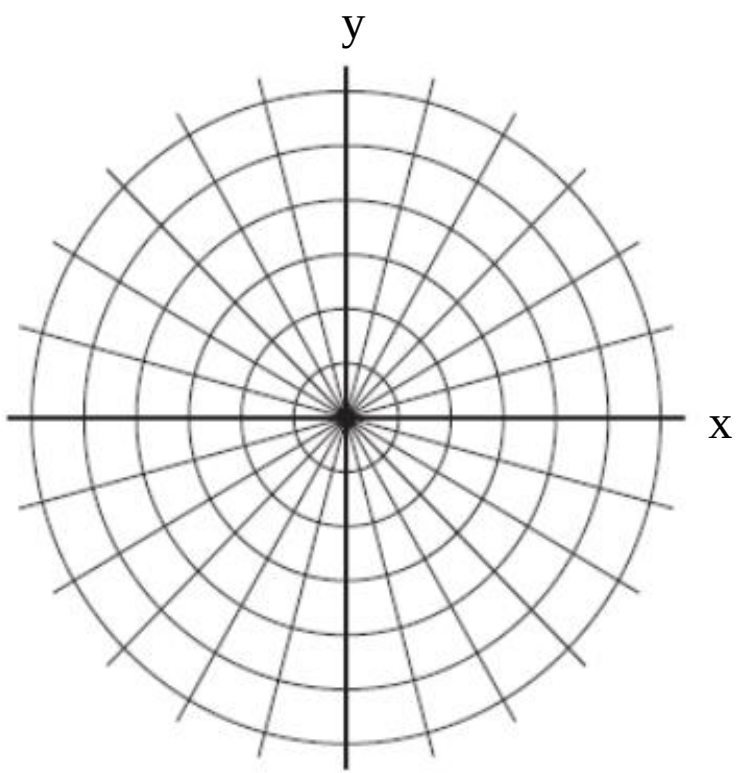
$$\frac{2\pi}{3} \le \theta \le \frac{5\pi}{6}$$

Rectangular Equation for the same graph:

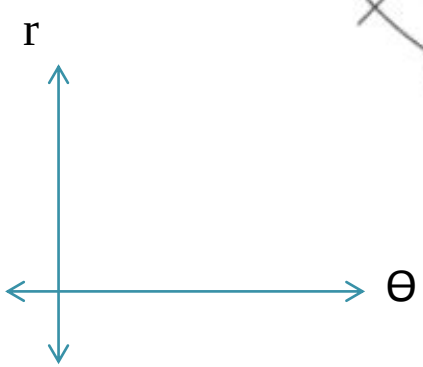
$$x^2 + (y-3)^2 = 9$$

Only need [0, π], this is the graphing domain

Graph r = 6 sin ϴ

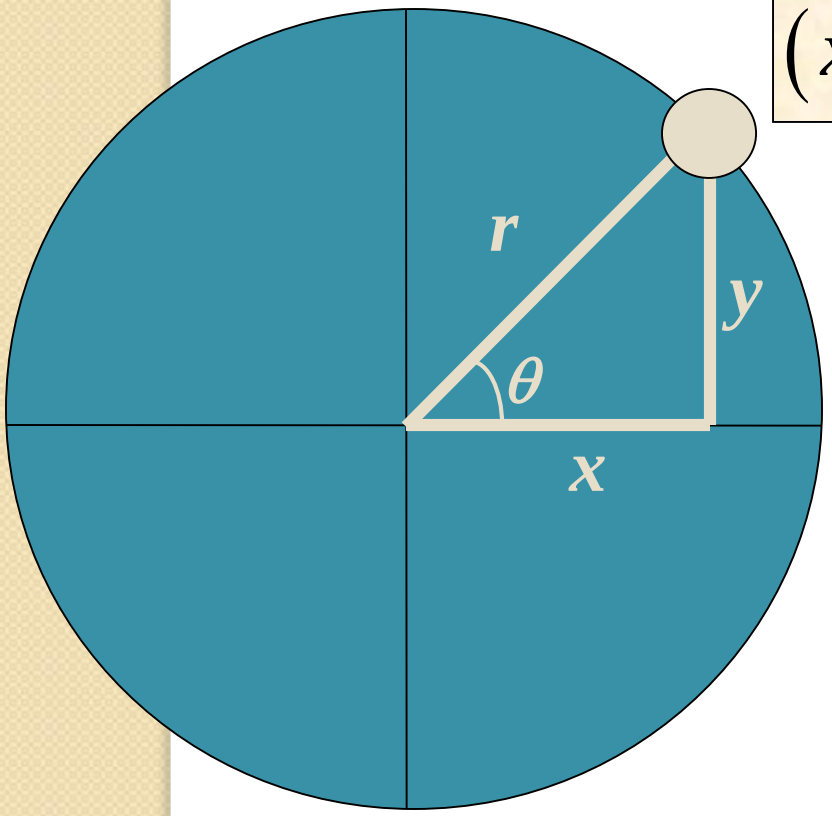


θ	r
0	
π/6	
π/3	
π/2	
2π/3	
5π/6	
π	
7π/6	
4π/3	
3π/2	
5π/3	
11π/6	
2π	



Period is 2π

How do we convert from polar to Cartesian?



$$(r, \theta)$$

$$(x, y)$$

$$\cos\theta = \frac{x}{r} \qquad x = r\cos\theta$$

$$\sin\theta = \frac{y}{r} \qquad y = r\sin\theta$$

Examples

Convert into Cartesian: (4, π/6)

$$x = r\ \cos\theta$$

$$x = 4\ \cos\frac{\pi}{6}$$

$$x = 4\left(\frac{\sqrt{3}}{2}\right)$$

$$x = 2\sqrt{3}$$

$$y = r\ \sin\theta$$

$$y = 4\ \sin\frac{\pi}{6}$$

$$y = 4\left(\frac{1}{2}\right)$$

$$y = 2$$

$$Answer: (2\sqrt{3}, 2)$$

Practice

Convert into Cartesian:

$$1)\ \left(4, \frac{7\pi}{3}\right)$$

$$2)\left(-3, \frac{5\pi}{4}\right)$$

$$3)\left(-5, \frac{\pi}{6}\right)$$

$$Answers:$$

$$1)\ \left(2, 2\sqrt{3}\right)$$

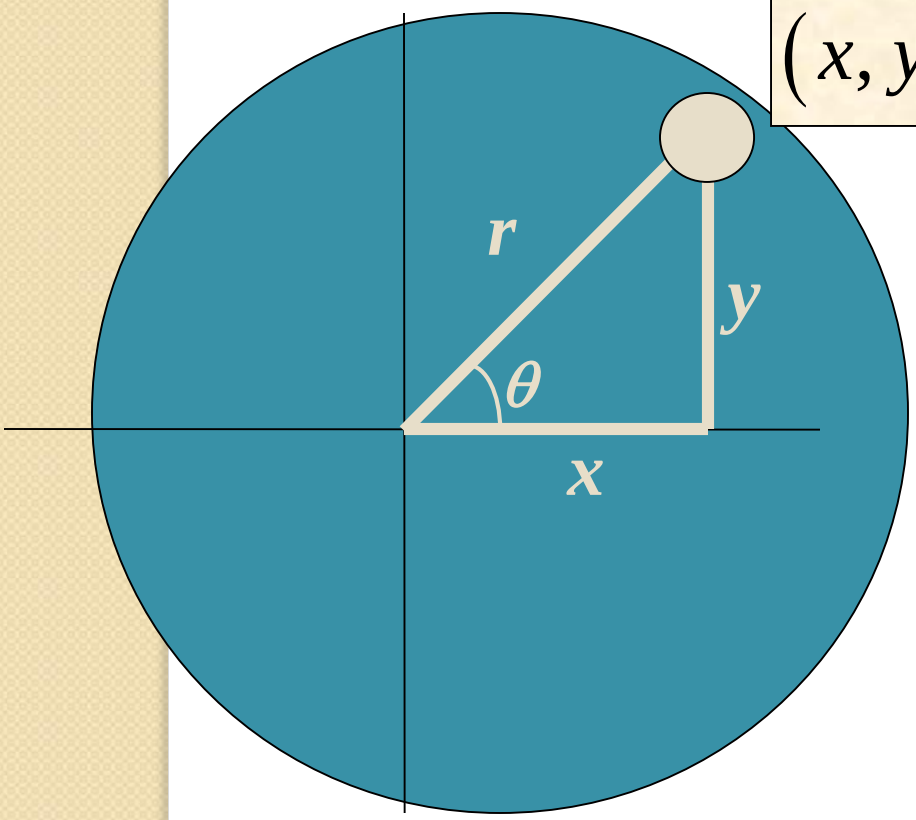
$$2)\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$$

$$3)\left(\frac{-5\sqrt{3}}{2}, -\frac{5}{2}\right)$$

How do we convert Cartesian back to polar?

$$(r, \theta)$$

$$(x, y)$$



$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Convert to Polar $\left(-3, \sqrt{3}\right)$

$$r^2 = x^2 + y^2$$

$$r^2 = (-3)^2 + \left(\sqrt{3}\right)^2$$

$$r^2 = 9 + 3$$

$$r = 2\sqrt{3}$$

$$\tan\theta = \frac{y}{x}$$

$$\tan\theta = \frac{\sqrt{3}}{-3}$$

$$\theta = \frac{5\pi}{6}$$

So one solution is: $\left(2\sqrt{3}, \frac{5\pi}{6}\right)$

But another co[illegible]: $\left(-2\sqrt{3}, -\frac{\pi}{6}\right)$

Remember, there are multiple conversions to polar! ☺

A note on Tangent

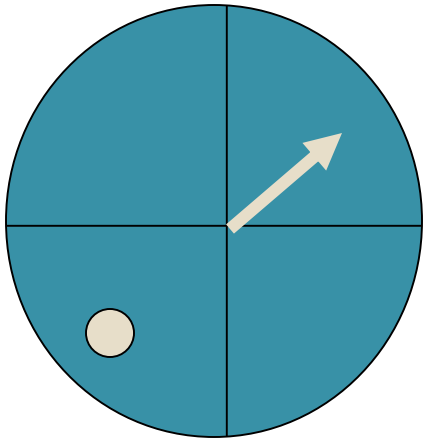
Convert (-3, -4) to polar coordinates

$$r = \sqrt{(-3)^2 + (-4)^2}$$
$$r = 5$$

$$\theta = \tan^{-1}\left(\frac{-4}{-3}\right) = 0.927$$

0.927 is a first quadrant angle

But (-3, -4) is a 3rd quadrant point



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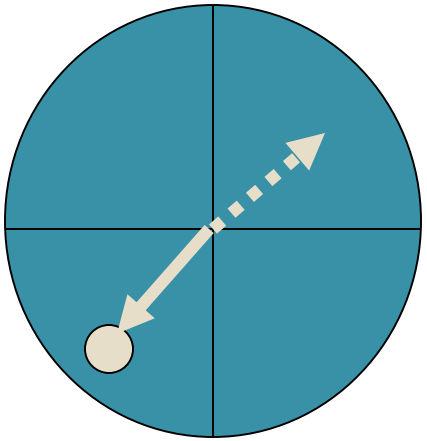

0.927 is a first quadrant angle

But (-3, -4) is a 3rd quadrant point

So we add π $\theta = 0.927 + \pi = 4.069$ (5, 4.069)

Or make r negative, (-5, 0.927)

Practice:
Convert into Polar Coordinates

$$1)\ \left(1, \sqrt{3}\right)$$

$$2)\ \left(1, -1\right)$$

$$3)\ \left(\sqrt{2}, -\sqrt{2}\right)$$

$$Answers:$$

$$(at\ least\ one\ of\ them)$$

$$1)\ \left(2, \frac{\pi}{3}\right)$$

$$2)\ \left(\sqrt{2}, -\frac{\pi}{4}\right)$$

$$3)\ \left(2, -\frac{\pi}{4}\right)$$

Converting Equations

Translate equations from one system to the other using the same formulas.

$$r = 2$$

$$r^2 = 4$$

$$x^2 + y^2 = 4$$

$$\theta = \frac{\pi}{3}$$

$$\tan^{-1}\frac{y}{x} = \frac{\pi}{3}$$

$$\frac{y}{x} = \tan\frac{\pi}{3}$$

$$\frac{y}{x} = \sqrt{3}$$

$$y = \sqrt{3}x$$

$$r = \sec\theta$$

$$r = \frac{1}{\cos\theta}$$

$$r\cos\theta = 1$$

$$x = 1$$

$$r = \frac{5}{\sin\theta - 2\cos\theta}$$

$$r\left(\sin\theta - 2\cos\theta\right) = 5$$

$$r\sin\theta - 2r\cos\theta = 5$$

$$y - 2x = 5$$

COMMON TIP: It may helpful to multiply both sides by "r"

$$r = -4\cos\theta$$

$$r^2 = -4r\cos\theta$$

$$x^2 + y^2 = -4x$$

$$x^2 + 4x + y^2 = 0$$

$$x^2 + 4x + \_\_\_\_\_ + y^2 = \_\_\_$$

$$x^2 + 4x + 4 + y^2 = 4$$

$$(x+2)^2 + y^2 = 4$$

Complete the square so you can tell what the graph looks like.

This is a circle whose center is (-2, 0) and radius is 2.