

## 1. Calculator Active.

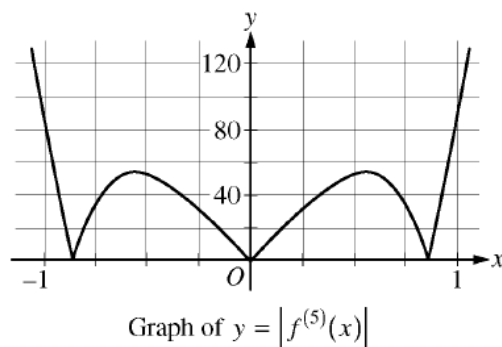
$x$	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\frac{488}{3}$	$\frac{448}{3}$	$\frac{584}{9}$
3	317	$\frac{753}{2}$	$\frac{1383}{4}$	$\frac{3483}{16}$	$\frac{1125}{16}$

Let  $h$  be a function having derivatives of all orders for  $x > 0$ . Selected values of  $h$  and its first four derivatives are indicated in the table above. The function  $h$  and these four derivatives are increasing on the interval  $1 \leq x \leq 3$ .

- Write the first-degree Taylor polynomial for  $h$  about  $x = 2$  and use it to approximate  $h(1.9)$ . Is this approximation greater than or less than  $h(1.9)$ ? Explain your reasoning.
- Write the third-degree Taylor polynomial for  $h$  about  $x = 2$  and use it to approximate  $h(1.9)$ .
- Use the Lagrange error bound to show that the third-degree Taylor polynomial for  $h$  about  $x = 2$  approximates  $h(1.9)$  with error less than  $3 \times 10^{-4}$ .

**Calculator Inactive.**

2. Let  $f(x) = \sin(x^2) + \cos x$ . The graph of  $y = |f^{(5)}(x)|$  is shown above.



- Write the first four nonzero terms of the Taylor series for  $\sin x$  about  $x = 0$ , and write the first four nonzero terms of the Taylor series for  $\sin(x^2)$  about  $x = 0$ .
- Write the first four nonzero terms of the Taylor series for  $\cos x$  about  $x = 0$ . Use this series and the series for  $\sin(x^2)$ , found in part (a), to write the first four nonzero terms of the Taylor series for  $f$  about  $x = 0$ .
- Find the value of  $f^{(6)}(0)$ .
- Let  $P_4(x)$  be the fourth-degree Taylor polynomial for  $f$  about  $x = 0$ . Using information from the graph of  $y = |f^{(5)}(x)|$  shown above, show that  $\left| P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right) \right| < \frac{1}{3000}$ .

## 3.

$$f(x) = \begin{cases} \frac{\cos x - 1}{x^2} & \text{for } x \neq 0 \\ -\frac{1}{2} & \text{for } x = 0 \end{cases}$$

The function  $f$ , defined above, has derivatives of all orders. Let  $g$  be the function defined by

$$g(x) = 1 + \int_0^x f(t) dt.$$

- Write the first three nonzero terms and the general term of the Taylor series for  $\cos x$  about  $x = 0$ . Use this series to write the first three nonzero terms and the general term of the Taylor series for  $f$  about  $x = 0$ .
- Use the Taylor series for  $f$  about  $x = 0$  found in part (a) to determine whether  $f$  has a relative maximum, relative minimum, or neither at  $x = 0$ . Give a reason for your answer.
- Write the fifth-degree Taylor polynomial for  $g$  about  $x = 0$ .
- The Taylor series for  $g$  about  $x = 0$ , evaluated at  $x = 1$ , is an alternating series with individual terms that decrease in absolute value to 0. Use the third-degree Taylor polynomial for  $g$  about  $x = 0$  to estimate the value of  $g(1)$ . Explain why this estimate differs from the actual value of  $g(1)$  by less than  $\frac{1}{6!}$ .