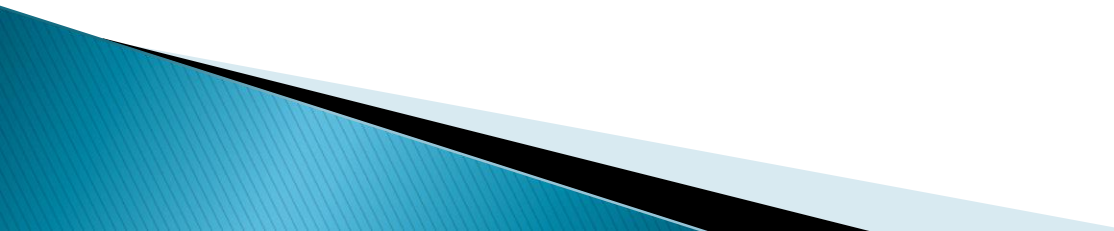


Section 2.3 Product and Quotient Formulas

- ▶ To be differentiable at a point a function must be continuous at the point and the left hand derivative must equal the right hand derivative.

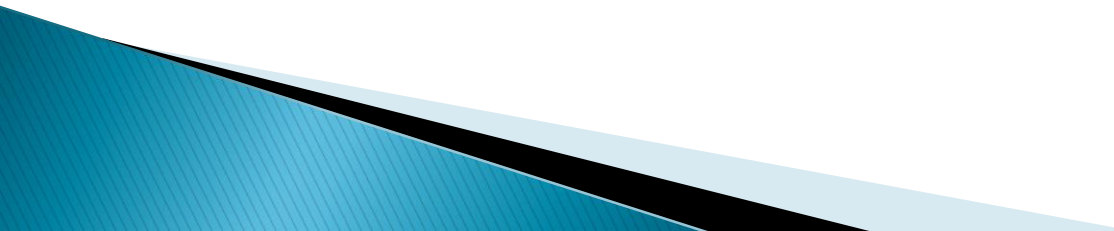
Is $f(x)$ differentiable at $x = 0$?

$$f(x) = \begin{cases} x^2 - 4, & x > 0 \\ 2x - 4, & x \leq 0 \end{cases}$$

- ▶ First check for continuity at $x = 0$.
 - ▶ Next, check derivatives at $x = 0$.
- 

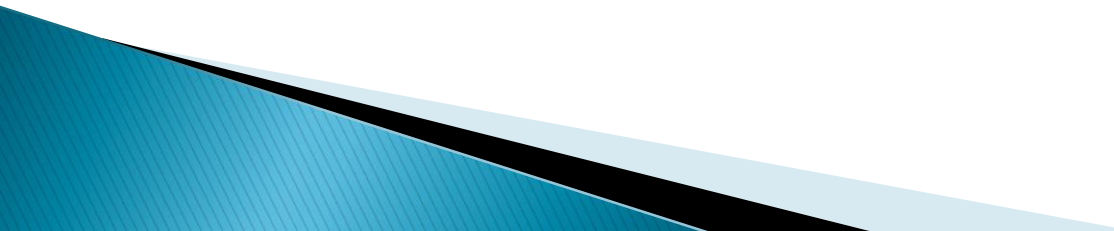
Is $g(x)$ differentiable at $x = 1$?

$$g(x) = \begin{cases} 8x - 3, & x \leq 1 \\ 4x^2 + 5, & x > 1 \end{cases}$$

- ▶ First check for continuity at $x = 1$.
 - ▶ Next, check derivatives at $x = 1$.
- 

Is $h(x)$ differentiable at $x = 3$?

$$h(x) = \begin{cases} x^2 - 4x + 8, & x \leq 3 \\ 2x - 1, & x > 3 \end{cases}$$

- ▶ First check for continuity at $x = 3$.
 - ▶ Next, check derivatives at $x = 3$.
- 

Find b and c so that $f(x)$ is differentiable at $x = 1$

$$f(x) = \begin{cases} 3x^2 + 4x, & x \leq 1 \\ 2x^3 + bx + c, & x > 1 \end{cases}$$

You try: Find a and b so that $f(x)$ is differentiable at $x = 2$

$$f(x) = \begin{cases} ax^2 + 10, & x < 2 \\ x^2 - 6x + b, & x \geq 2 \end{cases}$$

- ▶ You may think the product rule works like the sum rule for derivatives, but that is not true.

$y = x^3$ can be written $y = x * x^2$

$$y' = 3 x^2$$

$$y' = 1 * 2x$$

$$3 x^2 \neq 2x$$

Product Rule: $(f \cdot g)' = f \cdot g' + f' \cdot g$

▶ Example:

$$f(x) = (6x^3)(7x^4)$$

Product Rule: $(f \cdot g)' = f \cdot g' + f' \cdot g$

► Example:

$$\frac{d}{dx} [(x^2 + 3)(2x^3 + 5x)]$$

This rule will come in handy for non-polynomial functions such as $f(x) = x^2 \sin x$.

Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$

$$\frac{\text{Lo De Hi} - \text{Hi De Lo}}{(\text{Lo})^2}$$

Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$

Example: $y = \frac{x^2 + x - 2}{x^3 + 6}$