Section 2.3 Product and Quotient Formulas

- To be differentiable at a point a function must be continuous at the point and the left hand derivative must equal the right hand derivative.

Is f(x) differentiable at x = 0?

$$f(x) = \begin{cases} x^2 - 4, x > 0 \\ 2x - 4, x \le 0 \end{cases}$$

- First check for continuity at x = 0.

- Next, check derivatives at x = 0.

Is g(x) differentiable at x = 1?

$$g(x) = \begin{cases} 8x - 3, x \leq 1 \\ 4x^2 + 5, x > 1 \end{cases}$$

- First check for continuity at x = 1.

- Next, check derivatives at x = 1.

Is h(x) differentiable at x = 3?

$$h(x) = \begin{cases} x^2 - 4x + 8, x \leq 3 \\ 2x - 1, x > 3 \end{cases}$$

- First check for continuity at x = 3.

- Next, check derivatives at x = 3.

Find b and c so that f(x) is differentiable at x = 1

$$f(x) = \begin{cases} 3x^2 + 4x, x \le 1 \\ 2x^3 + bx + c, x > 1 \end{cases}$$

You try: Find a and b so that f(x) is differentiable at x = 2

$$f(x) = \begin{cases} ax^2 + 10, x < 2 \\ x^2 - 6x + b, x \geq 2 \end{cases}$$

- You may think the product rule works like the sum rule for derivatives, but that is not true.

$y = x^3$ can be written $y = x * x^2$

$y' = 3x^2$ $\qquad\qquad$ $y' = 1 * 2x$

$$3x^2 \neq 2x$$

Product Rule: $\boxed{(f \cdot g)' = f \cdot g' + f' \cdot g}$

- Example:

  $f(x) = (6x^3)(7x^4)$

Product Rule: $\boxed{(f \cdot g)' = f \cdot g' + f' \cdot g}$

- Example:

$$\frac{d}{dx}[(x^2+3)(2x^3+5x)]$$

This rule will come in handy for non-polynomial functions such as $f(x) = x^2 \sin x$.

Quotient Rule: $\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}}$

$$\frac{\text{Lo De Hi} - \text{Hi De Lo}}{(\text{Lo})^2}$$

Quotient Rule: $\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}}$

Example: $y = \frac{x^2 + x - 2}{x^3 + 6}$