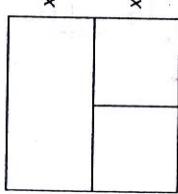


7. A farmer has 100 yards of fencing to form two identical rectangular pens and a third pen that is twice as long as the other two pens, as shown in the diagram below. All three pens have the same width, x . Which value of y produces the maximum total fenced area?



$$5x + 6y = 100 \quad \frac{6y = 100 - 5x}{6}$$

$$y = \frac{100 - 5x}{6}$$

$$\begin{aligned} 2xy + 2yx &= 4xy \\ H \times y &= A \text{ area} \\ 4x \left(\frac{100 - 5x}{6} \right) &= A \\ \frac{400x}{6} - \frac{20x^2}{6} &= A \\ A' = \frac{400}{6} - \frac{40}{6}x &= 0 \end{aligned}$$

$$\begin{aligned} 400 - 40x &= 0 \\ 400 &= 40x \\ 10 &= x \end{aligned}$$

118 ♦ CHAPTER 3

$$\begin{aligned} y &= \frac{100 - 5(10)}{6} \\ \frac{50}{6} &= \frac{25}{3} \end{aligned}$$

118 ♦ CHAPTER 3

8. For the function $f(x) = 12x^5 - 5x^4$, how many of the inflection points of the function are also extrema?

- (A) 3
(B) 2
(C) 1
(D) None

D

9. The position of an object moving along a straight line for $t \geq 0$ is given by $s_1(t) = t^3 + 2$, and the position of a second object moving along the same line is given by $s_2(t) = t^2$. If both objects begin at $t = 0$, at what time is the distance between the objects a minimum?

- (A) 2
(B) $\frac{50}{27}$
(C) $\frac{2}{3}$
(D) 0

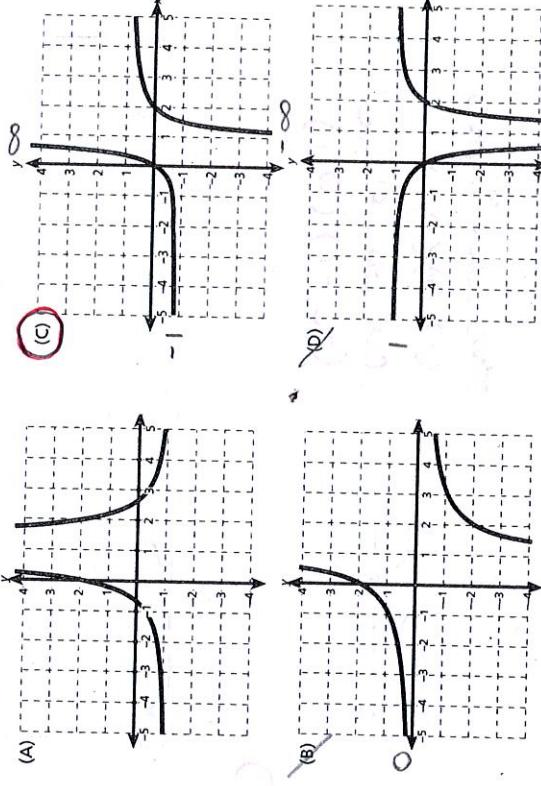
- B

$$\begin{aligned} \text{distance} &= t^3 + 2 - t^2 \\ d' &= 3t^2 - 2t = 0 \\ t(3t - 2) &= 0 \\ t = 0 \quad t &= \frac{2}{3} \end{aligned}$$

$$d'' = 6t - 2$$

$$d''(0) = -2 \quad d''\left(\frac{2}{3}\right) = 2$$

10. Given the following conditions for $f(x)$, which graph best illustrates $f(x)$? The domain of the function is the real numbers, but $x \neq 1$.
- $\lim_{x \rightarrow -\infty} f(x) = -1$; $\lim_{x \rightarrow 1^-} f(x) = \infty$; $\lim_{x \rightarrow 1^+} f(x) = -\infty$.
 $f'(x) > 0$ for all x where $x \neq 1$, and $f'(x)$ does not exist at $x = 1$.
 $f''(x) > 0$ for $x < 1$, $f''(x) < 0$ for $x > 1$, and $f''(x)$ does not exist at $x = 1$.



C

A calculator may be used for the following question.

11. Let $f(x)$ be a function such that $f'(x) = \ln x \cdot \cos x + \frac{\sin x}{x}$. In the interval $0 < x < 3$, the graph of $f(x)$ has a point of inflection nearest $x =$

- (A) 0.352
(B) 1.101
(C) 2.128
(D) 2.259

B

$$f'(x) = \ln x \cdot (-\sin x) + \frac{\cos x}{x} + \frac{\sin x - x \cos x}{x^2}$$

$$f''(x) =$$

$$t(3t - 2) = 0$$

$$t = 0 \quad t = \frac{2}{3}$$

$$d'' = 6t - 2$$

$$d''(0) = -2 \quad d''\left(\frac{2}{3}\right) = 2$$

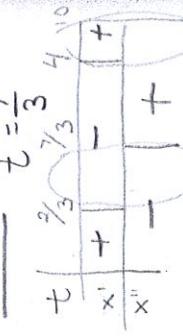
$$x' = 3t^2 - 14t + 8$$

- A calculator may not be used on the following questions.
- $x' = (3t-2)(t-4) = 0$
- Questions 12 and 13 refer to the following information:

For time $0 \leq t \leq 10$, a particle moves along the x -axis with position given by $x(t) = t^4 - 7t^2 + 8t + 5$.

12. During what time intervals is the speed of the particle increasing?

- (A) $4 < t \leq 10$ only
 (B) $0 \leq t < \frac{2}{3}$ and $\frac{7}{3} < t < 4$
 (C) $0 \leq t < \frac{2}{3}$ and $4 < t \leq 10$
 (D) $\frac{2}{3} < t < \frac{7}{3}$ and $4 < t \leq 10$

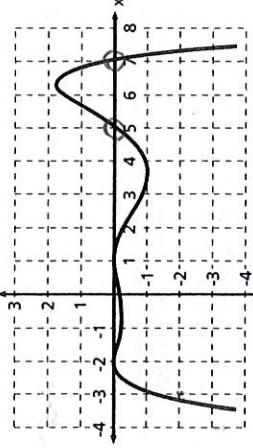


13. What is the position of the particle when it is farthest to the left?

- (A) -14
 (B) -11
 (C) $-\frac{47}{27}$
 (D) $\frac{203}{27}$

$$x(0) = 5$$

$$x(4) = 4^4 - 7(4)^2 + 8(4) + 5 = 101 - 112 + 32 + 5 = 101 - 112 = -11$$



14. Based on the graph of $g'(x)$ pictured above, how many points of inflection exist for the twice differentiable function $g(x)$ on the interval $-4 < x < 8$?

- (A) 4
 (B) 3
 (C) 2
 (D) 1

at $5, 7$

$z \in 0 \rightarrow$ sign change

$$f'' = -12x = 0$$

$x = 0$ Inflection pt

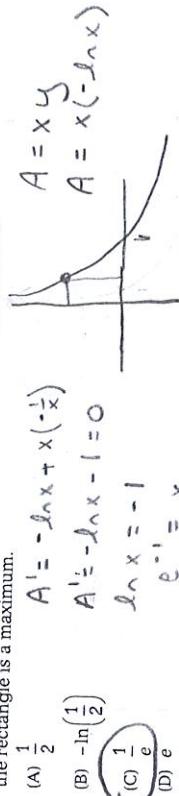
© 2017 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part, except for use as permitted in a license distributed with a certain product or service or otherwise on a password-protected website for classroom use.

$$g'' = |-12x| = 0, x = 0 \text{ undefined at } x = 2$$

c) $g'(x)$ is not defined at $x = 2$ b/c there is a sharp corner there

- d) f has IP at $x = 0$, $g(x)$ would have IP at $x = 0$

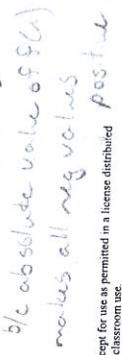
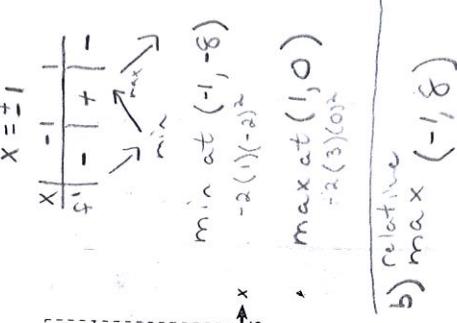
C) $-2 < x < 0$



FREE-RESPONSE QUESTION

This question does not require the use of a calculator.

1. The function $f(x)$ is defined as $f(x) = -2(x+2)(x-1)^2$ on the open interval $(-3, 3)$ as illustrated in the graph shown.
- (a) Determine the coordinates of the relative extrema of $f(x)$ in the open interval $(-3, 3)$.
- (b) Let $g(x)$ be defined as $g(x) = |f(x)|$ in the open interval $(-3, 3)$. Determine the coordinate(s) of the relative maxima of $g(x)$ in the open interval. Explain your reasoning.
- (c) For what values of x is $g'(x)$ not defined? Explain your reasoning.
- (d) Find all values of x for which $g(x)$ is concave down. Explain your reasoning.



b/c absolute value of f(x)

makes all negative values positive

© 2017 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part, except for use as permitted in a license distributed with a certain product or service or otherwise on a password-protected website for classroom use.

15. A rectangle is drawn in the first quadrant so that it has two adjacent sides on the coordinate axes and one vertex on the curve $y = -\ln(x)$. Find the x coordinate of the vertex for which the area of the rectangle is a maximum.

- (A) $\frac{1}{2}$
 (B) $-\ln\left(\frac{1}{2}\right)$
 (C) $\frac{1}{e}$
 (D) e^{-1}

C

D

B

C

$$C = 2\pi r$$

$$h = 12 \quad r = 4 \quad \frac{dC}{dt} = \frac{\pi}{4}$$

15. When the height of a cylinder is 12 cm and the radius is 4 cm, the circumference of the cylinder is increasing at a rate of $\frac{\pi}{4}$ cm/min, and the height of the cylinder is increasing four times faster than the radius. How fast is the volume of the cylinder changing?

(A) $\frac{\pi}{2}$ cm³/min

(B) 4π cm³/min

(C) 12π cm³/min

(D) 20π cm³/min

D

FREE-RESPONSE QUESTION

A calculator may be used for this question.

1. An isosceles triangle is inscribed in a semicircle, as shown in the diagram, and it continues to be inscribed as the semicircle changes size. The area of the semicircle is increasing at the rate of 1 cm²/sec when the radius of the semicircle is 3 cm.

(a) How fast is the radius of the semicircle increasing when the radius is 3 cm? Include units in your answer.

(b) How fast is the perimeter of the semicircle increasing when the radius is 3 cm? Include units in your answer.

(c) How fast is the area of the isosceles triangle increasing when the radius is 3 cm? Include units in your answer.

(d) How fast is the shaded region increasing when the radius is 3 cm? Include units in your answer.

c) $A = \frac{1}{2} \pi r^2$

b) $P = \frac{1}{2}(2\pi r) + d$

c) $A = \frac{1}{2}bh = \frac{1}{2}(2r)(r)$

$$A = r^2$$

$$\frac{dA}{dt} = 2r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2(3) \frac{1}{3}\pi$$

$$\frac{dA}{dt} = \frac{2}{3}\pi$$

$$\frac{dP}{dt} = \pi \left(\frac{1}{3}\pi\right) + 2\left(\frac{1}{3}\pi\right)$$

$$= \frac{1}{3} + \frac{2}{3}\pi$$

$$\frac{dP}{dt} = \frac{2}{3}\pi$$

$$\frac{dA}{dt} = \frac{2}{3}\pi^2/\text{sec}$$

d) $A = \frac{1}{4} \pi r^2 - \frac{1}{2} \Delta$

$$= \frac{1}{4} \pi r^2 - \frac{1}{2} r^2$$

$$A = \frac{1}{4} \pi r^2 - \frac{1}{2} r^2$$

$$\frac{dA}{dt} = \left(\frac{\pi}{4} - \frac{1}{2}\right) 2r \frac{dr}{dt}$$

$$= \left(\frac{1}{2} - \frac{1}{2}\right) 2r \frac{dr}{dt}$$

$$= 0$$

$$V = \pi r^2 h$$

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

$$\frac{dh}{dt} = 4 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\left(\frac{1}{8}\right) = \frac{1}{2}$$

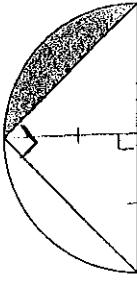
$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + \pi 2r \frac{dr}{dt} h$$

$$\frac{1}{8} = \frac{dr}{dt}$$

$$\frac{dV}{dt} = \pi(4)^2 \cdot \frac{1}{2} + \pi(2)(4)\left(\frac{1}{8}\right)12$$

$$= 8\pi + 12\pi$$

$$= 20\pi$$



FREE-RESPONSE QUESTION

	Solution	Possible points																
(a)	<p>Given: $f(-3) = 32$ and $f(3) = -40$</p> $f(x) = -2(x+2)(x-1)^2 = -2x^3 + 6x - 4$ $f'(x) = -6x^2 + 6 = -6(x^2 - 1) = 0, \text{ so } x = \pm 1.$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td>$-3 < x < -1$</td> <td>$-1 < x < 1$</td> <td>$1 < x < 3$</td> </tr> <tr> <td>$f'(x)$</td> <td>Positive</td> <td>Negative</td> <td>Positive</td> </tr> <tr> <td>$f(x)$</td> <td>Decreasing</td> <td>Increasing</td> <td>Decreasing</td> </tr> </table> <p>$f(1) = 0; f(-1) = -8$</p> <p>Therefore in the open interval $(-3, 3)$, the relative maximum for $f(x)$ occurs at the point $(1, 0)$ and the relative minimum for $f(x)$ occurs at the point $(-1, -8)$.</p>		$-3 < x < -1$	$-1 < x < 1$	$1 < x < 3$	$f'(x)$	Positive	Negative	Positive	$f(x)$	Decreasing	Increasing	Decreasing	$1: f'(x)$ $3: 1: x = \pm 1$ $1: \text{answer}$ $1: \text{and reason}$				
	$-3 < x < -1$	$-1 < x < 1$	$1 < x < 3$															
$f'(x)$	Positive	Negative	Positive															
$f(x)$	Decreasing	Increasing	Decreasing															
(b)	<p>For $g(x) = f(x)$, the relative maximum will occur at the point $(-1, 8)$. All values of the new function $g(x)$ are made positive with the application of the absolute value to $f(x)$, and the relative minimum of $f(x)$ becomes a relative maximum for $g(x)$.</p>	$2: \begin{cases} 1: \text{reasoning} \\ 1: (-1, 8) \end{cases}$ \vdots																
(c)	<p>Since $g(x) = \begin{cases} -2x^3 + 6x - 4, & x < -2 \\ 2x^3 - 6x + 4, & x > -2 \end{cases}$</p> <p>then $g'(x) = \begin{cases} -6x^2 + 6, & x < -2 \\ 6x^2 - 6, & x > -2. \end{cases}$</p> <p>$g'(x)$ is not defined at $x = -2$ because $\lim_{x \rightarrow -2^-} g'(x) = -18$ and $\lim_{x \rightarrow -2^+} g'(x) = 18$. A sharp turn occurs at $x = -2$ on the graph.</p>	$2: \begin{cases} 1: x = -2 \\ 1: \text{reason} \end{cases}$																
(d)	<p>Since $g''(x) = 12x = 0$ at $x = 0$ and $g''(x)$ is not defined at $x = -2$, the inflection points occur at $x = 0$ and $x = -2$. Therefore</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>Interval</td> <td>$-3 < x < -2$</td> <td>$-2 < x < 0$</td> <td>$0 < x < 3$</td> </tr> <tr> <td></td> <td>$g''(x) = -12x$</td> <td>$g''(x) = 12x$</td> <td>$g''(x) = 12x$</td> </tr> <tr> <td>Sign of $g''(x)$</td> <td>Positive</td> <td>Negative</td> <td>Positive</td> </tr> <tr> <td>$g(x)$</td> <td>Concave up</td> <td>Concave down</td> <td>Concave up</td> </tr> </table> <p>and $g(x)$ is concave down in the open interval $(-2, 0)$.</p>	Interval	$-3 < x < -2$	$-2 < x < 0$	$0 < x < 3$		$g''(x) = -12x$	$g''(x) = 12x$	$g''(x) = 12x$	Sign of $g''(x)$	Positive	Negative	Positive	$g(x)$	Concave up	Concave down	Concave up	$2: \begin{cases} 1: f''(x) = 0 \\ 1: \text{interval} \\ 1: \text{and reason} \end{cases}$
Interval	$-3 < x < -2$	$-2 < x < 0$	$0 < x < 3$															
	$g''(x) = -12x$	$g''(x) = 12x$	$g''(x) = 12x$															
Sign of $g''(x)$	Positive	Negative	Positive															
$g(x)$	Concave up	Concave down	Concave up															

FREE-RESPONSE QUESTION

Solution	Possible points
(a) $A = \frac{\pi}{2} r^2$ $\frac{dA}{dt} = \pi r \frac{dr}{dt}$ $1 = 3\pi \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{3\pi} \approx 0.106 \text{ cm/sec}$	2: { 1: $\frac{dA}{dt}$ 1: answer }
(b) $p = \pi r + 2r$ $\frac{dp}{dt} = (\pi + 2) \frac{dr}{dt} = \frac{\pi + 2}{3\pi} \approx 0.546 \text{ cm/sec}$	2: { 1: $\frac{dp}{dt}$ 1: answer }
(c) $A = \frac{1}{2} \text{base} \times \text{height} = \frac{1}{2}(2r)r = r^2$ $\frac{dA}{dt} = 2r \frac{dr}{dt} = \frac{6}{3\pi} = \frac{2}{\pi} \approx 0.637 \text{ cm}^2/\text{sec}$	2: { 1: $\frac{dA}{dt}$ 1: answer }
(d) $A = \text{area of } \frac{1}{4} \text{ of the circle minus}$ $\text{the area of } \frac{1}{2} \text{ of the triangle.}$ $A = \frac{1}{4}\pi r^2 - \frac{1}{2}r^2$ $\frac{dA}{dt} = \left(\frac{\pi}{2}r - r\right) \frac{dr}{dt} = \frac{\frac{3\pi}{2} - 3}{3\pi} = \frac{3\pi - 6}{6\pi} \approx 0.182 \text{ cm}^2/\text{sec}$	2: { 1: $\frac{dA}{dt}$ 1: answer }
	1: { units for a, b, c, and d }

(a), (b), (c), (d) (*Calculus for AP* 1st ed. pages 189–193; EU 2.3; LO 2.3C; EK 2.3C2; MPAC 2,3,4,5)