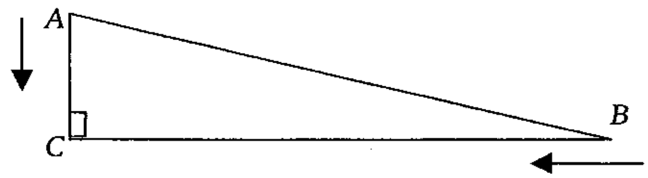
Day 2 Review Applications of Derivatives

❖ CHAPTER 2

8. In right triangle ΔABC, point A is moving along a leg of the right triangle toward point C at a rate of $\frac{1}{2}$ cm/sec and point B is moving toward point C at a rate of $\frac{1}{3}$ cm/sec along a line containing the other leg of the right triangle, as illustrated in the triangle shown below. What is the rate of change in the area of ΔABC, with respect to time, at the instant when $AC = 15$ cm and $BC = 20$ cm?



(A) -0.0833 cm²/sec
(B) -0.4167 cm²/sec
(C) -0.8333 cm²/sec
(D) -7.5 cm²/sec

6 ❖ CHAPTER 3

MULTIPLE-CHOICE QUESTIONS

A calculator may not be used on the following questions.

1. Let M represent the absolute maximum of $f(x)$ in an interval.
   Let R represent a root of $f(x)$ in the given interval.
   Let m represent the absolute minimum of $f(x)$ in the interval.
   If $f(x) = x^3 - 3x^2$, then which of the following is true over the closed interval $-3 \le x \le 1$?
   (A) M and R occur at a critical point and m occurs at an endpoint.
   (B) M and m occur at critical points.
   (C) M, m, and R occur at endpoints of the given interval.
   (D) M occurs at an endpoint, whereas m and R occur at a critical point.

2. What value of c in the open interval (0, 4) satisfies the Mean Value Theorem for $f(x) = \sqrt{3x+4}$?
   (A) 0
   (B) $\frac{3}{5}$
   (C) $\frac{5}{3}$
   (D) 2

3. If $f'(x) = \dfrac{x^2(x+1)}{(x-1)^{\frac{1}{3}}}$, then on which interval(s) is the continuous function $f(x)$ increasing?

(A) $(-\infty,-1)\cup(1,\infty)$

(B) $(-\infty,0)\cup(1,\infty)$

(C) $(-\infty,-1)\cup(0,\infty)$

(D) $(1,\infty)$

4. The points of inflection for $f(x)$ are at $x = p_1$ and $x = p_2$. Which of the following is (are) true?

I. The points of inflection for $f(x-a)$ are at $x = p_1 + a$ and $x = p_2 + a$.

II. The points of inflection for $bf(x)$ are at $x = b\,p_1$ and $x = b\,p_2$.

III. The points of inflection for $f(cx)$ are at $x = \dfrac{p_1}{c}$ and $x = \dfrac{p_2}{c}$.

(A) I only

(B) II only

(C) I and II only

(D) I and III only

5. $\lim\limits_{x\to 0} \dfrac{\sin 4x}{x^2+8x} =$

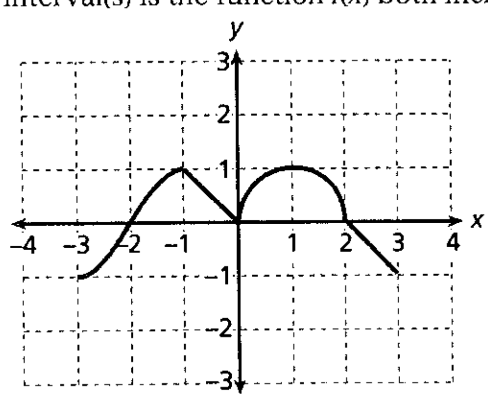
(A) 0

(B) $\dfrac{1}{2}$

(C) 1

(D) ∞

6. The graph of $f'(x)$ is given below for $x \in [-3,3]$. On which interval(s) is the function $f(x)$ both increasing and concave up?



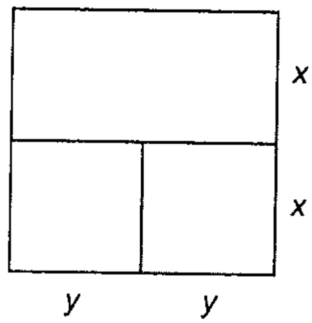
(A) $(-2,2)$

(B) $(-2,0)\cup(0,2)$

(C) $(-3,-2)$

(D) $(-2,-1)\cup(0,1)$

7. A farmer has 100 yards of fencing to form two identical rectangular pens and a third pen that is twice as long as the other two pens, as shown in the diagram below. All three pens have the same width, x. Which value of y produces the maximum total fenced area?



(A) $\frac{25}{2}$
(B) 10
(C) $\frac{100}{11}$
(D) $\frac{25}{3}$

8. For the function $f(x) = 12x^5 - 5x^4$, how many of the inflection points of the function are also extrema?
(A) 3
(B) 2
(C) 1
(D) None

9. The position of an object moving along a straight line for $t \geq 0$ is given by $s_1(t) = t^3 + 2$, and the position of a second object moving along the same line is given by $s_2(t) = t^2$. If both objects begin at $t = 0$, at what time is the distance between the objects a minimum?
(A) 2
(B) $\frac{50}{27}$
(C) $\frac{2}{3}$
(D) 0

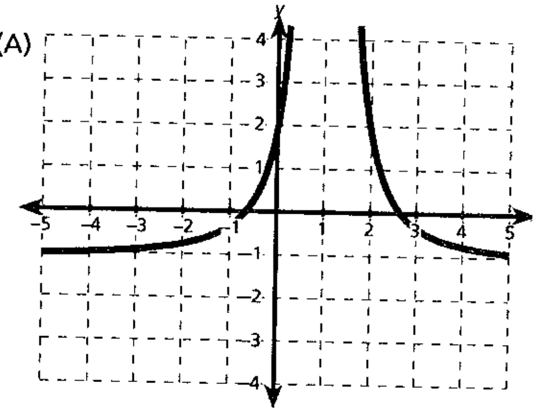
10. Given the following conditions for $f(x)$, which graph best illustrates $f(x)$?

$f(x)$: The domain of the function is the real numbers, but $x \neq 1$;
$\lim\limits_{x \to -\infty} f(x) = -1$; $\lim\limits_{x \to 1^-} f(x) = \infty$; $\lim\limits_{x \to 1^+} f(x) = -\infty$.

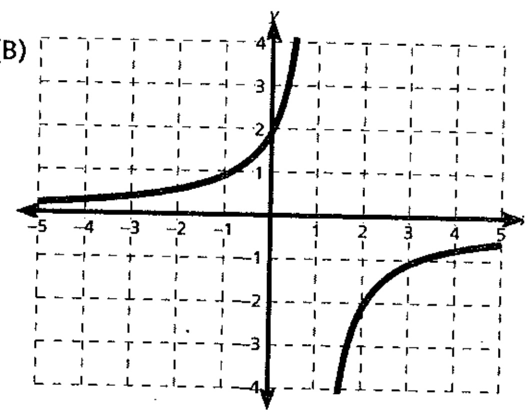
$f'(x) > 0$ for all x where $x \neq 1$, and $f'(x)$ does not exist at $x = 1$.

$f''(x) > 0$ for $x < 1$, $f''(x) < 0$ for $x > 1$, and $f''(x)$ does not exist at $x = 1$.

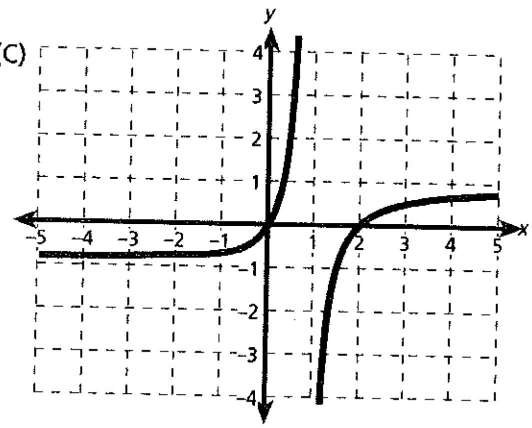
(A)



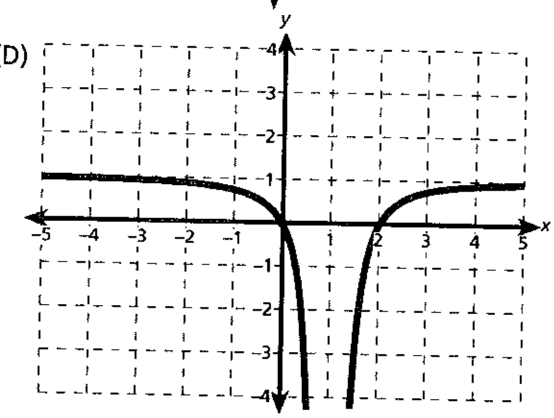
(B)



(C)



(D)



A calculator may be used for the following question.

11. Let $f(x)$ be a function such that $f'(x) = \ln x \cdot \cos x + \dfrac{\sin x}{x}$. In the interval $0 < x < 3$, the graph of $f(x)$ has a point of inflection nearest $x =$

(A) 0.352
(B) 1.101
(C) 2.128
(D) 2.259

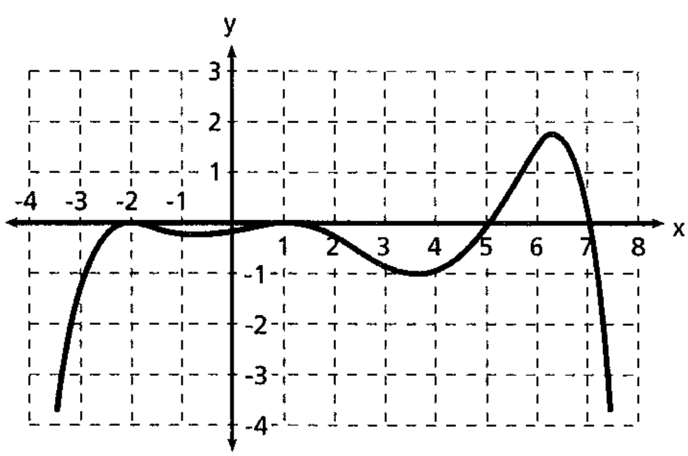
A calculator may not be used on the following questions.

Questions 12 and 13 refer to the following information:

For time $0 \le t \le 10$, a particle moves along the x-axis with position given by $x(t) = t^3 - 7t^2 + 8t + 5$.

12. During what time intervals is the speed of the particle increasing?
(A) $4 < t \le 10$ only
(B) $0 \le t < \frac{2}{3}$ and $\frac{7}{3} < t < 4$
(C) $0 \le t < \frac{2}{3}$ and $4 < t \le 10$
(D) $\frac{2}{3} < t < \frac{7}{3}$ and $4 < t \le 10$

13. What is the position of the particle when it is farthest to the left?
(A) -14
(B) -11
(C) $-\frac{47}{27}$
(D) $\frac{203}{27}$



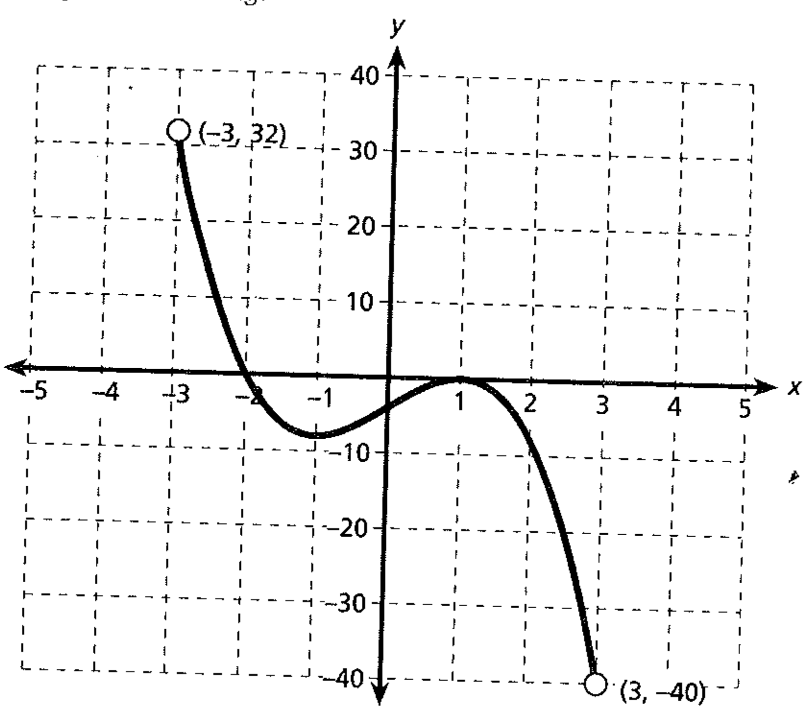
14. Based on the graph of $g''(x)$ pictured above, how many points of inflection exist for the twice differentiable function g(x) on the interval $-4 < x < 8$?
(A) 4
(B) 3
(C) 2
(D) 1

15. A rectangle is drawn in the first quadrant so that it has two adjacent sides on the coordinate axes and one vertex on the curve $y = -\ln(x)$. Find the x coordinate of the vertex for which the area of the rectangle is a maximum.
   (A) $\frac{1}{2}$
   (B) $-\ln\left(\frac{1}{2}\right)$
   (C) $\frac{1}{e}$
   (D) e

FREE-RESPONSE QUESTION

This question does not require the use of a calculator.

1. The function $f(x)$ is defined as $f(x) = -2(x+2)(x-1)^2$ on the open interval (–3, 3) as illustrated in the graph shown.
   (a) Determine the coordinates of the relative extrema of $f(x)$ in the open interval (–3, 3).
   (b) Let $g(x)$ be defined as $g(x) = |f(x)|$ in the open interval (–3, 3). Determine the coordinate(s) of the relative maxima of $g(x)$ in the open interval. Explain your reasoning.
   (c) For what values of x is $g'(x)$ not defined? Explain your reasoning.
   (d) Find all values of x for which $g(x)$ is concave down. Explain your reasoning.



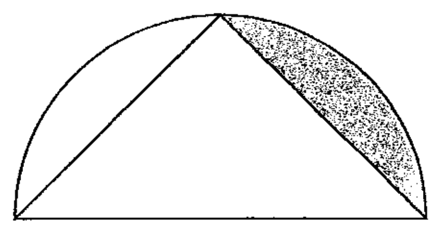
$f(x)$

15. When the height of a cylinder is 12 cm and the radius is 4 cm, the circumference of the cylinder is increasing at a rate of $\frac{\pi}{4}$ cm/min, and the height of the cylinder is increasing four times faster than the radius. How fast is the volume of the cylinder changing?
    (A) $\frac{\pi}{2}\ \text{cm}^3/\text{min}$
    (B) $4\pi\ \text{cm}^3/\text{min}$
    (C) $12\pi\ \text{cm}^3/\text{min}$
    (D) $20\pi\ \text{cm}^3/\text{min}$

FREE-RESPONSE QUESTION

A calculator may be used for this question.

1. An isosceles triangle is inscribed in a semicircle, as shown in the diagram, and it continues to be inscribed as the semicircle changes size. The area of the semicircle is increasing at the rate of 1 cm²/sec when the radius of the semicircle is 3 cm.



   (a) How fast is the radius of the semicircle increasing when the radius is 3 cm? Include units in your answer.
   (b) How fast is the perimeter of the semicircle increasing when the radius is 3 cm? Include units in your answer.
   (c) How fast is the area of the isosceles triangle increasing when the radius is 3 cm? Include units in your answer.
   (d) How fast is the shaded region increasing when the radius is 3 cm? Include units in your answer.