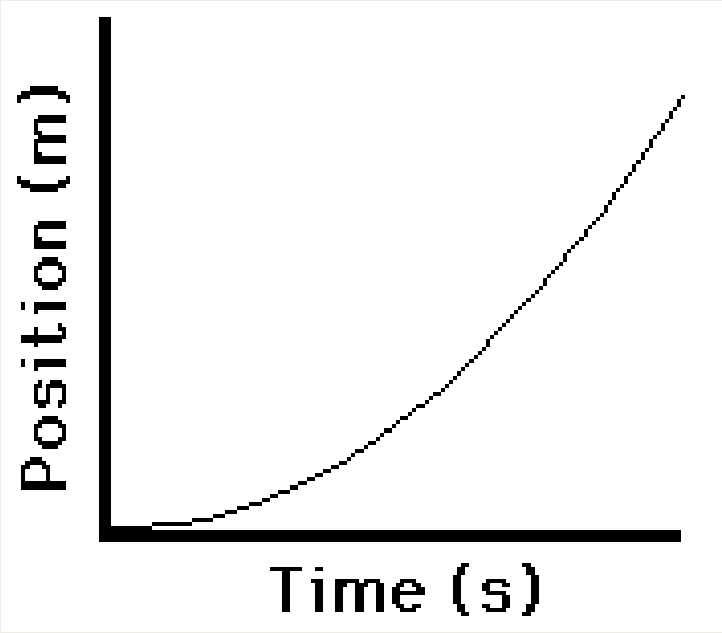
HIGHER ORDER DERIVATIVES AND MOTION

Section 2.3

Motion



This is the position function s(t). It shows the **displacement** (distance from starting point) as a function of time. Units for s(t) are meters.

$$\text{Velocity} = \frac{change\ in\ position}{change\ in\ time}$$

Average velocity is the slope between two points.

Instantaneous velocity is velocity at a specific time which would be the derivative at a point: $v(t) = s'(t)$.

The units for velocity are m/s.

- Velocity shows both speed and direction.

Positive velocity means moving forwards or upwards.

Negative velocity means moving backwards or down.

- Speed is the absolute value of velocity.

$$\text{Acceleration} = \frac{change\ in\ velocity}{change\ in\ time}$$

- **Average acceleration** is the slope between two points of velocity.
- **Instantaneous acceleration** is acceleration at a specific time which would be the derivative at a point: $\mathrm{a(t)} = v'(t) = s''(t)$.
- The units for acceleration are m/s^2.

$$\text{Jerk} = \frac{change\ in\ acceleration}{change\ in\ time}$$

- **Average jerk** is the slope between two points of acceleration.
- **Instantaneous jerk** is jerk at a specific time which would be the derivative at a point:

$$\text{j(t)} = \text{a}'(\text{t}) = v''(t) = s'''(t).$$

- The units for acceleration are m/s^3.

Example: The position of a particle is given by $s(t) = t^3 - 6t^2 + 9t$, with time in seconds and position in meters.

A. Find acceleration at 4 seconds.

$$s(t) = t^3 - 6t^2 + 9t$$

$$v(t) = s'(t) = 3t^2 - 12t + 9$$

$$a(t) = v'(t) = 6t - 12$$

$$a(4) = 6(4) - 12 = 12 \ m/s^2$$

B. When is the particle not moving?

Particle is not moving when $v(t) = 0$

$$v(t) = s'(t) = 3t^2 - 12t + 9 = 0$$

$$t^2 - 4t + 3 = 0$$

$$(t - 1)(t - 3) = 0$$

$$t = 1 \ or \ 3$$

Particle is not moving at 1 sec and 3 sec

Example: On the moon, an arrow is shot upwards at a velocity of 58m/s. Its height after t seconds is given by $h(t) = 58t - 0.83t^2$.

What is its velocity when it hits the ground?

The height when the arrow hits the ground is zero, so find when $h(t) = 0$.

$$h(t) = 58t - 0.83t^2 = 0$$

$$t(58 - 0.83t) = 0$$

$$t = 0 \; or \; \frac{58}{0.83}$$

Arrow started on the ground, then returned to the ground at 58/0.83 seconds.

$$v(t) = h'(t) = 58 - 1.66t$$

$$v\left(\frac{58}{0.83}\right) = 58 - 1.66\left(\frac{58}{0.83}\right) = -0.58 \text{ m/s}$$

Velocity is negative because the arrow was traveling down in a negative direction. The speed of the arrow was 0.58 m/s.