Section 9.3 Integral Test and p-series

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Integrals are a sum of infinite rectangles under a curve, so they should be related to infinite series.

Integral Test

If f(x) is positive, continuous and decreasing for $x \ge 1$ and $a_n = f(n)$, then

if
$$\int_{1}^{\infty} f(x) dx$$
 converges, then $\sum_{n=1}^{\infty} a_n$ converges
if $\int_{1}^{\infty} f(x) dx$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges



Example: $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$ Converge or Diverge?

Nth term test for Divergence?

Telescopic Series?

Geometric Series?

Is the function positive, continuous and decreasing for $x \ge 1$?

Then try integration.



Example: $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$ Converge or Diverge?

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Is the function positive, continuous and decreasing for $x \ge 1$?

Then try integration.

A series in the form
$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$
 is called a **p-series**.

When
$$p = I$$
, $\sum_{n=1}^{I} \frac{1}{n}$

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This series is called the **harmonic series**.

(In music, the wavelengths of overtones of a vibrating string form a harmonic series.)

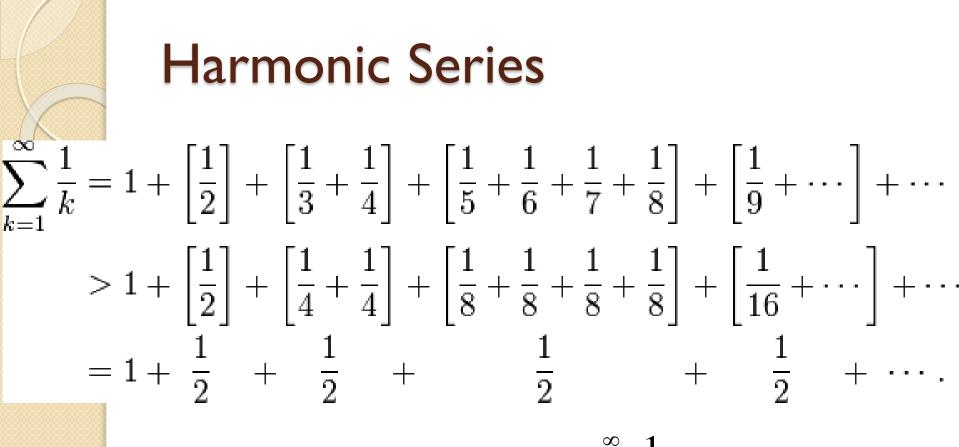
We looked at p-integrals earlier this year. What values of p caused the integral to converge?

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In p-series,
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If p > I, the series converges.

If $p \leq I$, the series diverges.

This was proven by integration earlier.



Basically this implies that $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$

Examples: Converge or Diverge? $\sum_{n=1}^{\infty} \frac{1}{n^3} \qquad \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \qquad \sum_{n=1}^{\infty} \frac{1}{3^n}$

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