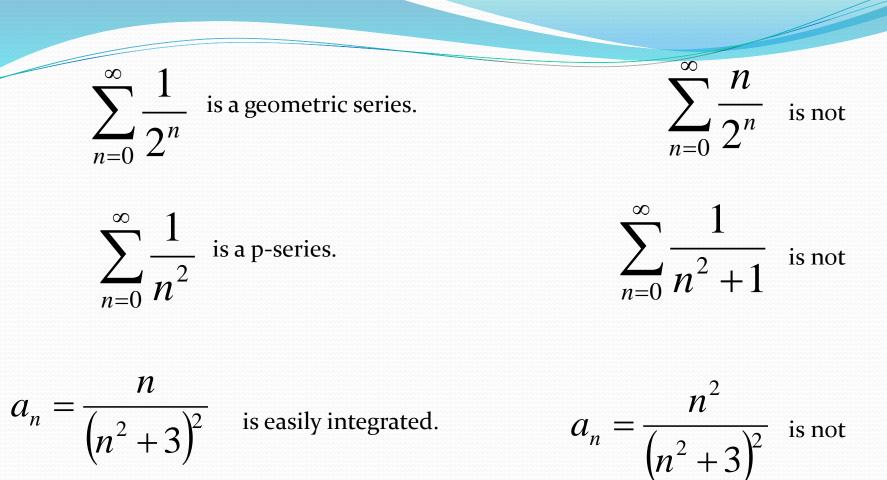
Section 9.4 Comparison Tests



 $(n^2 + 3)^2$ T 1 1 1 1

To check the convergence of an unknown series, compare it to a series you know.

Direct Comparison Test (DCT)

For series with positive terms, let $o \le a_n \le b_n$ for all n

If
$$\sum b_n$$
 converges, then $\sum a_n$ (something smaller) converges
If $\sum a_n$ diverges, then $\sum b_n$ (something larger) diverges

Examples

Does
$$\sum_{n=1}^{\infty} \frac{1}{3^n + 2}$$
 converge or diverge?

Examples

Does
$$\sum_{n=1}^{\infty} \frac{5 \ln n}{2n}$$
 converge or diverge?

Try: $\sum_{n=1}^{\infty} \frac{1}{3n^2 + 2}$

 $\sum_{n=1}^{\infty} \frac{3^n}{4^n + 5}$

 $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$

Limit Comparison Test (LCT)

Given two series $\sum a_n$ and $\sum b_n$. Let's compare the terms of each using limits.

If
$$\lim_{n \to \infty} \frac{a_n}{b_n} = 1$$
, then $a_n \approx b_n$.

If
$$\lim_{n \to \infty} \frac{a_n}{b_n} = c$$
 , then $a_n \approx cb_n$

So if b_n converges, a_n also converges. If b_n diverges, a_n also diverges.

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The constant doesn't change the convergence of the series since it can be factored out.

Limit Comparison Test (LCT)

Given two series $\sum a_n$ and $\sum b_n$. Let's compare the terms of each using limits.

If
$$\lim_{n \to \infty} \frac{a_n}{b_n} = 0$$
, then $a_n < b_n$.

If
$$\lim_{n \to \infty} \frac{a_n}{b_n} = \infty$$
, then $a_n > b_n$

We are back to the Direct Comparison Test.

So if b_n converges, the smaller a_n also converges. If b_n diverges, we don't know about a_n .

If b_n diverges, the larger a_n also diverges. If b_n converges, we don't know about $a_{n.}$

Limit Comparison Test (LCT)

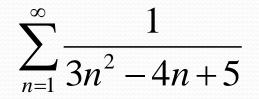
For series with positive terms,

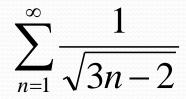
If $\lim_{n \to \infty} \frac{a_n}{b_n} =$ **a finite, nonzero constant**, then the series

behave the same, both diverge or both converge.

So what about $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$?

More examples: Converge or Diverge.





More examples: Converge or Diverge.

$$\sum_{n=1}^{\infty} \frac{n^2 - 10}{4n^5 + n^3}$$



Practice: p.573 #1-25odd, 27-34