#### Section 9.6 Ratio and Root Tests

Last 2 tests!!!

Geometric series have a constant ratio between terms. Other series have ratios that are not constant. We will look at the ratio between consecutive terms to determine convergence.

**10.7.5** THEOREM (*Ratio Test for Absolute Convergence*). Let  $\sum u_k$  be a series with nonzero terms and suppose that

$$\rho = \lim_{k \to +\infty} \frac{|u_{k+1}|}{|u_k|}$$

- (a) If  $\rho < 1$ , then the series  $\sum u_k$  converges absolutely and therefore converges.
- (b) If  $\rho > 1$  or if  $\rho = +\infty$ , then the series  $\sum u_k$  diverges.
- (c) If  $\rho = 1$ , no conclusion about convergence or absolute convergence can be drawn from this test.

#### Use this test for exponential or factorial expressions.

# Example

Use the ratio test for absolute convergence to determine whether the series converges.

(a) 
$$\sum_{k=1}^{\infty} (-1)^k \frac{2^k}{k!}$$
 (b)  $\sum_{n=1}^{\infty} \frac{n^2 2^{n+1}}{3^n}$ 

## Example 5

Use the ratio test for absolute convergence to determine whether the series converges.

(a) 
$$\sum_{k=1}^{\infty} (-1)^k \frac{2^k}{k!}$$
 (b)  $\sum_{k=1}^{\infty} (-1)^k \frac{(2k-1)!}{3^k}$   
(a)  $\rho = \lim_{k \to +\infty} \frac{|u_{k+1}|}{|u_k|} = \lim_{k \to +\infty} \frac{2^{k+1}}{(k+1)!} \cdot \frac{k!}{2^k} = \lim_{k \to +\infty} \frac{2}{k+1} = 0 < 1$   
converges absolutely

## Example

Use the ratio test for absolute convergence to determine whether the series converges.

- k

(a) 
$$\sum_{k=1}^{\infty} (-1)^k \frac{2^k}{k!}$$
 (b)  $\sum_{k=1}^{\infty} (-1)^k \frac{(2k-1)!}{3^k}$   
(b)  $\rho = \lim_{k \to +\infty} \frac{|u_{k+1}|}{|u_k|} = \lim_{k \to +\infty} \frac{(2k+1)!}{3^{k+1}} \cdot \frac{3^k}{(2k-1)!} = \lim_{k \to +\infty} \frac{(2k+1)(2k)}{3} = +\infty$   
diverges

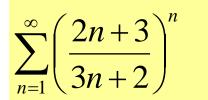
And finally... the Root Test

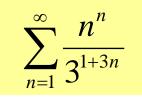
Let {a<sub>n</sub>} be a sequence and assume that the following limit exists:

If 
$$\lim_{n \to \infty} \sqrt[n]{|a_n|} < 1$$
 then  $\sum_{n=1}^{\infty} a_n$  converges absolutely
If  $\lim_{n \to \infty} \sqrt[n]{|a_n|} > 1$ , then  $\sum_{n=1}^{\infty} a_n$  diverges
If  $\lim_{n \to \infty} \sqrt[n]{|a_n|} = 1$ , the Ratio Test is INCONCLUSIVE

Use this test for expressions raised to the nth power.

#### Examples





Try

