Section 9.8 Power Series

Series Part 2!

Geometric Series

Example: $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1}$ Converges to when

a = r =

Both a and r are constants.

But what if they weren't?

Series as functions

Example: $f(x)=\sum_{n=1}^{\infty} 3(x)^{n-1}$

n is the counter that creates each term in a single series.

x is a variable that creates different series.

$$f\left(\frac{1}{4}\right)=$$

$$f(2)=$$

When will $f(x)=\sum_{n=1}^{\infty} 3(x)^{n-1}$ converge?

Vocabulary

<u>Interval of Convergence</u>: the interval of the domain values, x, for which the resulting series will converge

<u>Center of Convergence</u>: midpoint of interval of convergence

<u>Radius of Convergence</u>: distance from the center of convergence to the endpoints of the interval of convergence

$$f(x) = \sum_{n=1}^{\infty} 3(x)^{n-1}$$

Interval of convergence: (-1, 1)

Center of Convergence: x = 0

Radius of convergence: R = 1

$$f(x) = \sum_{n=1}^{\infty} 3(2x-6)^{n-1}$$

$$f(x) = \sum_{n=1}^{\infty} 5(2x+4)^{n-1}$$

Interval of convergence:

Center of Convergence:

Radius of convergence:

Interval of convergence:

Center of Convergence:

Radius of convergence:

Geometric Series are subsets of Power Series

Theses are geometric series

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} + \cdots$$

a and r are constants.

$$f(x) = \sum_{n=1}^{\infty} ax^{n-1} = a + ax + ax^2 + ax^3 + \cdots + ax^{n-1} + \cdots$$

a is a constant, r is a variable

In a power series, the coefficients do not have to be constant.

Power Series

General form usually starts with n = 0

$$f(x)=\sum_{n=0}^{\infty} C_n x^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \cdots + C_n x^n + \cdots$$

This power series is centered at zero.

$$f(x)=\sum_{n=0}^{\infty} C_n (x-a)^n = C_0 + C_1 (x-a) + C_2 (x-a)^2 + C_3 (x-a)^3 + \cdots + C_n (x-a)^n + \cdots$$

This power series is centered at a.

For each power series find C_n and the center of convergence

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n!}$$

$$\sum_{n=0}^{\infty} \frac{x^n}{2\ln n}$$

In a geometric series, you use the condition that in order to converge, $|r|<1$.

Geometric series diverge at 1 and – 1.

For a general power series, you must use the ratio test (or occasionally the root test).

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|<1$$

Once you find the interval of convergence, **<u>you must check the endpoints</u>** because the ratio test is inconclusive if the limit equals 1 or -1.

There are 3 possible outcomes.

1. The series converges over some finite interval, centered at a, $|x-a|<R$, R is the radius of convergence.
   (Diverges for $|x-a|>R$)

2. The series converges only at the center point, x = a. (Diverges everywhere else) R = 0.

3. Converges for all real numbers. R = ∞ .

Case 1:

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$$

Case 2:

$$\sum_{n=0}^{\infty} n!\,x^n$$

Case 3:

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$$

This is a type of Bessel Function. Among other applications, Bessel Functions are use to model vibrating surfaces like drum heads and heat conduction of circular surfaces.

You try:

$$\sum_{n=0}^{\infty} \frac{nx^n}{2^{n+1}}$$

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