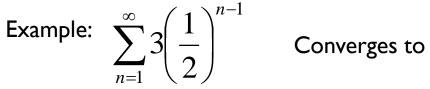
Section 9.8 Power Series

Series Part 2!

0

Geometric Series

a =



r =

when

Both a and r are constants.

But what if they weren't?



Example:
$$f(x) = \sum_{n=1}^{\infty} 3(x)^{n-1}$$

n is the counter that creates each term in a single series.

 \mathbf{x} is a variable that creates different series.

$$f\left(\frac{1}{4}\right) =$$

$$f(2) =$$

When will
$$f(x) = \sum_{n=1}^{\infty} 3(x)^{n-1}$$
 converge?

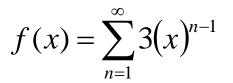


Vocabulary

<u>Interval of Convergence</u>: the interval of the domain values, x, for which the resulting series will converge

<u>Center of Convergence</u>: midpoint of interval of convergence

<u>Radius of Convergence</u>: distance from the center of convergence to the endpoints of the interval of convergence



Interval of convergence: (-1, 1)

Center of Convergence: x = 0

Radius of convergence: R = I

$$f(x) = \sum_{n=1}^{\infty} 3(2x - 6)^{n-1}$$

$$f(x) = \sum_{n=1}^{\infty} 5(2x+4)^{n-1}$$

Interval of convergence:

Center of Convergence:

Radius of convergence:

Interval of convergence:

Center of Convergence:

Radius of convergence:

Geometric Series are subsets of Power Series

Theses are geometric series

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \dots + ar^{n-1} + \dots$$

a and r are constants.

$$f(x) = \sum_{n=1}^{\infty} ax^{n-1} = a + ax + ax^2 + ax^3 + \dots + ax^{n-1} + \dots$$

a is a constant, r is a variable

In a power series, the coefficients do not have to be constant.

Power Series

General form usually starts with n = 0

$$f(x) = \sum_{n=0}^{\infty} C_n x^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \dots + C_n x^n + \dots$$

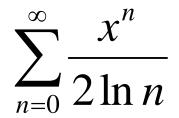
This power series is centered at zero.

$$f(x) = \sum_{n=0}^{\infty} C_n (x-a)^n = C_0 + C_1 (x-a) + C_2 (x-a)^2 + C_3 (x-a)^3 + \dots + C_n (x-a)^n + \dots$$

This power series is centered at a.

For each power series find C_n and the center of convergence

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n!}$$



In a geometric series, you use the condition that in order to converge, |r| < 1 .

Geometric series diverge at I and -I.

For a general power series, you must use the ratio test (or occasionally the root test).

 $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$

Once you find the interval of convergence, <u>you must check</u> <u>the endpoints</u> because the ratio test is inconclusive if the limit equals 1 or -1.

There are 3 possible outcomes.

- I. The series converges over some finite interval, centered at a, |x-a| < R, R is the radius of convergence. (Diverges for |x-a| > R)
- 2. The series converges only at the center point, x = a. (Diverges everywhere else) R = 0.
- 3. Converges for all real numbers. $R = \infty$.



Case I: $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$ n $\overline{n=1}$



Case 2:

 $\sum_{n=0}^{\infty} n! x^n$



Case 3: $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n} (n!)^2}$

This is a type of Bessel Function. Among other applications, Bessel Functions are use to model vibrating surfaces like drum heads and heat conduction of circular surfaces.



You try:

 $\sum_{n=0}^{\infty} \frac{nx^n}{2^{n+1}}$

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